B.E. / B.Tech. Computer Science & Engineering (Model Curriculum) Semester-III

SE101CS - Applied Mathematics-III

P. Pages : 3 **GUG/S/25/13801**

Time : Three Hours * 2 2 5 4 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. All questions are compulsory.
3. Non programmable calculator is permitted.

1. a) Prove that $Z\left\{n^p\right\} = -z\frac{d}{dz}Z\left\{n^{p-1}\right\}$, **8**

where p is any positive integer and hence deduce that $Z\{n\} = \frac{z}{(z-1)^2}$ and

$Z\left\{n^2\right\} = \frac{z(z+1)}{(z-1)^3}$.

b) If $F\{z\} = \frac{2z^2+3z+12}{(z-1)^4}$, find f_2. **8**

OR

2. a) Find Z-Transform of $\sin(3n+5)$ and $\cos(3n+5)$. **8**

b) By using convolution theorem, find $Z^{-1}\left\{\frac{z^2}{(z-1)(z-3)}\right\}$. **8**

3. a) Verify whether the following vectors are linearly dependence. If dependent, find the relation between them. $X_1 = (1,2,3), X_2 = (3,-2,1), X_3 = (1,-6,5)$ **8**

b) Find the eigen values and eigen vector of $A = \begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}$. **8**

OR

4. a) Find the inverse of the matrix if the following matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. **8**

b) Test the consistency and $2x-y-z=2, x+2y+z=2, 4x-7y-5z=2$ **8**

5. a) Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}$ represented by **8**

$A^8+A^7-18A^6-39A^5+A^4-18A^3-40A^2+2I$.

b) If $s=\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}$, find the matrix e^m by Sylvester's theorem. **8**

OR

6. a) Use matrix method to solve the D.E. $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+6y=0, y(0)=1, y'(0)=2$. **8**

b) Solve $\dfrac{d^2x}{dt^2}+4\dfrac{dx}{dt}-12x=0, x(0)=0, x'(0)=8$ by matrix method. **8**

7. a) Find the distribution function for r.v. X whose density function is **8**

$$f(x)=\begin{cases}\dfrac{x}{2}, & 0\le x\le 2\\ 0, & \text{otherwise}\end{cases}$$

b) The joint probability function of two discrete random variables X and Y is given by **8**

$$f(x)=\begin{cases}c(2x+y), & 0\le x\le 2, 0\le y\le 3\\ 0, & \text{otherwise}\end{cases}$$

OR

8. a) Let X and Y be continuous r.v. having joint density function **8**

$$f(x,y)=\begin{cases}c(x^2+y^2), & 0\le x\le 1, 0\le y\le 1\\ 0, & \text{otherwise}\end{cases}$$

i) determine $P\left(\dfrac{1}{4}<X<3/4\right)$ (ii) $(Y<1/2)$

ii) find marginal distribution functions of X and Y

iii) Determine whether X and Y are independent

b) Can the function, $f(x)=\begin{cases}c(1-x^2), & 0\le x\le 1\\ 0, & \text{otherwise}\end{cases}$ be a distribution function? **4**

c) Let X be a random variable giving the number of aces in a random draw of four cards from a pack of 52 cards. Find the probability function and the distribution function for X. **4**

9. a) Find mean, variance and moment generating function for exponential distribution **8**

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

b) Find (i) the range (ii) the semi-interquartile range and (iii) the mean deviation for The r.v. having density function $f(x) = \begin{cases} 2(1-x), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$. **8**

OR

10. a) Let $f(x) = \begin{cases} 6(x - x^2), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ **8**

Be the density function of r.v. X find (i) mode (ii) Median of the distribution.

b) Let X and Y be random variables having joint density function **8**

$$f(x, y) = \begin{cases} \dfrac{3x(x+y)}{5}, & 0 \le x \le 1, 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) $E(X), E(Y)$ (ii) $E(x^2 + y^2)$ (iii) $E(x^2)$ (iv) $E(y^2)$.

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